Dear Shurik,

I have been asking people about the questions you raised in your letter. Each day I postpone writing to you, thinking that I could send more complete answers the next day. But since it is getting close to the time ~~thatxyoux~~ when ~~youxarrixe~~ you will be coming to visit, I'll send off these rather incomplete comments. The end of May , beginning of June seem to be fine for me, Deligne. As for me I must lecture on mondays, but ~~mxxkx ot~~ other days are free. Do you want a room at Ormaille?

The answer to Question (a) seems to be no. Here is a sketch of an example that Deligne made up. Let H be the group generated by x,y with the relations $x^3=y^3=1$, and $xyx^{-1}y^{-1}$ is central. Let C be the (infinite cyclic) group generated by $xyx^{-1}y^{-1}$. So: $1\to C\to H\to\Gamma\to 1$ (where $\Gamma = Z/3Z\times Z/3Z$).

Now let G be the universal cover of $SL_2 R$, and identify the infinite cyclic group C with the center of G. By a (noncommutative group analogue of) pushout, Deligne constructs an extension

$$\begin{tikzcd}
1 \arrow[r] & C \arrow[r] \arrow[d] & H \arrow[r] \arrow[d, dashed] & \Gamma \arrow[r] \arrow[d, "="] & 1 \\
1 \arrow[r] & G \arrow[r, dashed] & E \arrow[r] & \Gamma \arrow[r] & 0
\end{tikzcd}$$

which doesn't split.

The answer to (b) seems to be yes for Lie groups, but beyond that I don't know. E.g. whether a contractible group (∞-dim'l) can have elements of finite order.

The question about the cone is answered negatively. Namely, there is an involution of B^3 with a circle fixed point set in the boundary S^2 (indeed a tame circle) yet whose fixed point set in the interior is wild. (a Bing-type example)

The question about finite subgroups of the Teichmuller group seems to be related to recent work of Thurston, but I don't yet know the full story there. Best, et à bientôt

~~Please~~ (t.s.v.p)

Barry

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I just checked with Siebenmann and a Teichmuller space expert named Kravitz — The situation is this: (1) It is unknown whether every finite subgroup $\Gamma \subseteq$ Teich. group operates on an algebraic curve. But
(2) It is true for $\Gamma =$ finite solvable gp. +
(3) True for genus $= 2$

Concerning the hypersurface $Y \subset X$ and finite autom. n of X which is identity on X — Siebenmann says that this is certainly solvable using Smith Theory (i.e. $n = 1$).