Voorschoten, May 16 1967.
Palestrinalaan 11.

Dear Grothendieck,

I am trying to work out your letter with indications. However I have many difficulties and my progress is very slow. Usually I could find out from your manuscripts for what purpose you developped things in such or such a way, but this time I doubt whether I have grasped the essential points. I can understand your set-up only against the lemma of Abhyankar as background, namely instead of giving directly the covering r(X) (in your notation) of S, you give the ''behaviour'' of the covering over other coverings, namely the s(M) , which are of a particular type; apparantly the behaviour over all these s(M) together gives better information then the r(X) alone. Is this intuitively a reasonable way of thinking over your new set-up?

My main difficulty is that I don't see that for a Galois covering the extra structure is uniquely determined. First the concept of Galois covering itself: After the abstract definition and some variations you write: (I assume that you have a copy) On peut enfin l'expliciter en revenant à la definition de $\mathrm{Rev}^{\underline{R}}(S)$, et en exigeant que les revêtements des s(M) correspondants à X soient principaux de groupe G. Now in all your examples there is also a group working on the s(M), for instance in example 1 a group $\mathfrak{g}$ working on Z, and this group acts also on the Galois covering X. It seems to me that one must require that the actions of G and $\mathfrak{g}$ commute; is this correct? On the same and the next page of your letter you explain (for example 1) that the functor r (geometric realization) is fully faithful (restricted to Galois coverings) under certain conditions; this I don't understand at all (and ~~makes~~ this makes me feel very uncertain, since I expect that this is an important point if one wants to see what is going on).

corresponding with

Some other points: in the final formulation of example 3 you introduce a sheme of groups G(M); ~~with this formulation~~ do the objects in example 3a ,with this formulation, consist out of couples M=(a, n) and G(M) = μ_n ? Also you write: $\mathrm{Hom}_{S'}(M',M)$ est vide sauf si $\underline{n}' \geq \underline{n}$, auquel cas....etc....., it seems to me that this should be ....sauf si $\underline{n}' = \underline{m}.\underline{n}$, auquel cas..... and where $\underline{m} = (m_i)_{i\in I}$ is a a set of positive integers.

I realize that I cause you a lot of trouble and work by asking every time for further explanation and that my assistance in writing the exposé is not very large. Therefore I suggest that you answer this letter only in case it is clear from my questions that I have really a misconception of your new set-up (and it would not surprise me if I have...). Otherwise I shall try to work out your theory as good as I can and make a preliminary manuscript which we can discuss when I am at Bures (I plan to come June 2). In any case, I hope to complete the definite version during my stay in Bures.

With kind regards,

Sincerely yours,

J.P. Murre

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https://grothendieck.umontpellier.fr
Université de Montpellier, UMR CNRS 5149, Case courrier 051, Place Eugène Bataillon
34095 Montpellier cedex 5 - France