Faire copie pour Jouanolou

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Dear Grothendieck,

I have been reading recently some notes from the IHES (one of your seminars, I suppose) entitled "Exposé V. Systèmes projectifs J-adiques, par J.P. Jouanolou". Could you tell me (for purposes of reference) to which seminar they belong, and whether there is a more recent edition?

Also I have some comments and questions. The most serious question is that I do not understand the proof of the "Theorem of Shih", A.3.1 on page 46. It seems to me that the lemma on p. 45 is valid only for $r > p$ (cf EGA 0_{III} 13.5.5.2), and so ~~the~~ the equivalence of statements (i) and (ii) of the theorem is not clear. (I do agree, however, with the weaker statement "(resp....)" in parentheses). This casts doubt on the following theorem A.3.2 on p.47, and also on the earlier Prop. 5.3.1 on p.40. These difficulties would disappear if one worked throughout in the category pro-A of projective systems with the usual equivalence, rather than in the ~~category~~ finer category P_{AR}.

Other comments: p.42 line -1. Isn't the spectral sequence actually biregular in case of a finite filtration?

p.47 line -1. I think the hypothesis should be stated for n and $n+1$, instead of n and $n-1$ (cf EGA 0_{III} 11.1.10).

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p. 38 I believe the artinian hypothesis in this proposition is unnecessary, if one assumes that X and Z are both AR-J-adic noetherian. To be precise, let

$$0 \to X \to Y \to Z \to 0$$

be an exact sequence of projective systems of A-modules, such that $J^{n+1}X_n = J^{n+1}Y_n = J^{n+1}Z_n = 0$ for all n, and assume X and Z are AR-J-adic noetherian. Then Y is also AR-J-adic noetherian.

Indeed, taking $\varprojlim$ we get an exact sequence

$$0 \to M' \to M \to M'' \to 0$$

of A-modules, with M', M'' finite type, so M is also finite type. Let $\mathcal{A}$ be the projective system $(A/J^{n+1})_n$. Then by Artin-Rees, we have an exact sequence

$$0 \to M' \otimes \mathcal{A} \to M \otimes \mathcal{A} \to M'' \otimes \mathcal{A} \to 0$$

in the category P_{AR} of projective systems up to translation. There is a natural map of this sequence to the first. The two outside arrows are isomorphisms, so $M \otimes \mathcal{A} \xrightarrow{\approx} Y$, which shows that Y is AR-J-adic.

I came to these questions while working on my paper "Affine Duality and Cofiniteness" which I am finally preparing for publication. I will send you a copy when it is ready.

Sincerely yours,

R. Hartshorne

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